

CRITERIUL MODULULUI

- Dacă $H_p(s) = \frac{k_p}{(T_z s + 1) \cdot \prod_{k=1}^n (T_k s + 1)}$ (funcția de transfer la procesului)

unde T_z, T_k sunt constante de timp cu următoarea proprietate

$$\begin{cases} T_k \leq 10 \text{ sec}, \forall k \in \{1, \dots, n\} \\ T_z \leq 0,1 \cdot \min_k T_k \end{cases}$$

$T_k \rightarrow$ constante de timp dominante

$T_z \rightarrow$ constanta de timp parazită

- Atunci se poate alege regulatorul

$$H_R(s) = \frac{\prod_{k=1}^n (T_k s + 1)}{2 \cdot k_p \cdot T_z s}$$

care asigură performanțele

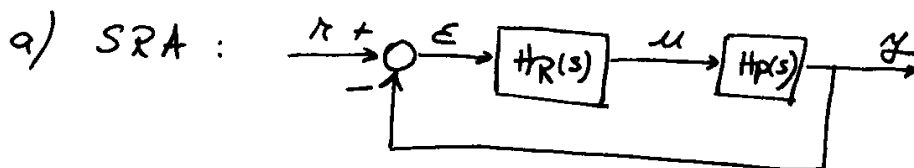
$$\begin{cases} \sigma = 4,3\% \\ t_t = 8 \cdot T_z \\ \epsilon_{st} = 0 \\ \epsilon_n = 2 \cdot T_z \end{cases}$$

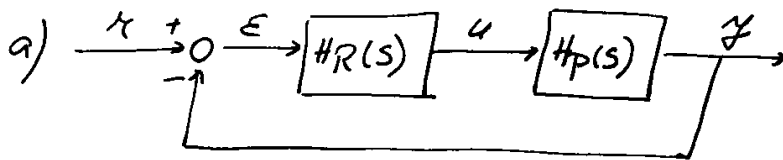
Exemplu: $H_p(s) = \frac{100}{(0,1s+1)(2s+1)(8s+1)}$

Se cere: a) Structura SRA

b) legea de reglare astfel încât să se

obțină $\sigma \leq 5\%$, $\epsilon_{st} = 0$, $t_t \leq 1,2 \text{ sec}$, $\epsilon_n \leq 0,3$





b) $e_{H_0} \geq e_{H_P} \Rightarrow e_{H_0} \geq 2 \Rightarrow$ Alegem $H_0(s)$ de forma unui sistem de ordinul II.

$$H_0(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$e_{st} = 0 \Rightarrow \lim_{s \rightarrow 0} H_0(s) = 1 \Rightarrow \frac{\omega_n^2}{\omega_n^2} = 1 \quad \checkmark$$

$$\sigma \leq 5\% \Rightarrow \text{Alegem } \zeta = 0,7 \text{ (pentru } \zeta = 0,7 \text{ avem } \sigma \approx 4,3\%)$$

$$\Rightarrow \sigma \approx 4,3\% < 5\% \quad \checkmark$$

$t_t \leq 17 \text{ ms} \Rightarrow$ Avem $\zeta = 0,7 \in [0,6; 0,8]$ deci putem aproxima t_t în modul următor: $t_t \approx \frac{4}{\zeta\omega_n}$

$$\Rightarrow \frac{4}{\zeta\omega_n} \leq 17 \text{ ms} \Rightarrow \frac{4}{0,7\omega_n} \leq 17 \text{ ms} \Rightarrow$$

$$\Rightarrow \omega_n \geq \frac{4}{11,9} \Rightarrow \omega_n \geq 0,33$$

Calculăm $H_d(s)$: $H_d(s) = \frac{H_0(s)}{1 - H_0(s)} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$

Calculăm $H_P(s)$: $H_P(s) = H_d(s) \cdot \frac{1}{H_R(s)} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \cdot (2s+1)(5s+1)$

$$\Rightarrow H_P(s) = \frac{\omega_n^2}{2\zeta\omega_n \cdot s \left(\frac{1}{2\zeta\omega_n} s + 1 \right)} \cdot (2s+1)(5s+1) =$$

$$= \frac{\omega_n}{2\zeta} \cdot \frac{(2s+1)(5s+1)}{s \left(\frac{1}{2\zeta\omega_n} s + 1 \right)}$$

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$$\text{Caz I: } \frac{1}{2\zeta\omega_u} = 5 \Rightarrow \omega_u = \frac{1}{10\zeta} = \frac{1}{7} = 0,14 < 0,33 \Rightarrow$$

$\Rightarrow$ nu vom pune

$$\text{Caz II: } \frac{1}{2\zeta\omega_u} = 2 \Rightarrow \omega_u = \frac{1}{4\zeta} = \frac{1}{2,8} \approx 0,36 > 0,33 \Rightarrow$$

$$\Rightarrow \boxed{\omega_u = 0,36} \quad (\omega_u = \frac{1}{4\zeta})$$

$$\Rightarrow H_R(s) = \frac{1}{4\zeta} \cdot \frac{1}{2\zeta} \cdot \frac{5s+1}{s} = \underbrace{\frac{5}{4}}_{k_R} \left(1 + \underbrace{\frac{1}{5s}}_{T_i} \right)$$

$\Rightarrow H_R(s)$ este un regulator de tip PI implementabil cu:

$$\begin{cases} k_R = \frac{5}{4} \\ T_i = 5 \text{ ms} \end{cases}$$

Deoarece $0,5 < 50 \rightarrow$ se poate neglija

$$\Rightarrow H_p(s) \approx \frac{50}{50s+1}$$

b) răspuns aperiodic $\Rightarrow \zeta = 0 \Rightarrow$ Alegem $H_0(s)$ de forma unui sistem de ordin I

$$H_0(s) = \frac{k}{Ts+1} \quad (e_{H_0} \geq e_{H_p} \Rightarrow 1 \geq 1 \rightarrow \text{admirat})$$

Pentru a avea $e_{st} = 0 \Rightarrow \lim_{s \rightarrow 0} H_0(s) = 1 \Rightarrow \boxed{k=1}$

Pentru a avea $t_t \leq 100 \text{ ms} \Rightarrow 3 \cdot T \leq 100 \text{ ms} \Rightarrow T \leq 33 \text{ ms}$

$\Rightarrow$ Alegem $\boxed{T=30 \text{ ms}}$

Atunci ~~$H_R(s)$~~ $H_0(s) = \frac{1}{30s+1} \Rightarrow H_d(s) = \frac{H_0(s)}{1-H_0(s)} = \frac{1}{30s}$

$\Rightarrow$ funcția de transfer a regulatorului $H_R(s) = H_d(s) \cdot \frac{1}{H_p(s)}$

$$\Rightarrow H_R(s) = \frac{1}{30s} \cdot \frac{50s+1}{50} = \frac{1}{30} \cdot \frac{50s+1}{50s} = \underbrace{\frac{1}{30}}_{k_R} \left(1 + \underbrace{\frac{1}{50s}}_{T_i} \right)$$

$\Rightarrow H_R(s)$ este un regulator de tip PI cu:

$$\begin{cases} k_R = \frac{1}{30} \\ T_i = 50 \text{ ms} \end{cases} \rightarrow \text{este implementabil}$$

Exemplul 2: Se dă $H_p(s) = \frac{1}{(2s+1)(5s+1)}$

Se cere: a) Structura SRA

b) legea de reglare ca: $\zeta \leq 5\%$, $t_t \leq 17 \text{ ms}$,
 $e_{st} = 0$

METODA POLI-ZEROURI

Se impune o $H_0(s)$ (funcție de transfer a sistemului în buclă închisă) care să satisfacă performanțele impuse.

Dacă avem $H_0(s)$, atunci putem calcula $H_d(s)$ (funcția de transfer a sistemului în buclă deschisă)

$$H_d(s) = \frac{H_0(s)}{1 - H_0(s)}$$

Știind $H_d(s)$ putem calcula $H_R(s)$ (funcția de transfer a regulatorului)

$$H_d(s) = H_R(s) \cdot H_P(s) \Rightarrow H_R(s) = H_d(s) \cdot \frac{1}{H_P(s)}$$

Dorim ca $H_R(s)$ să fie implementabil $e_{H_R} \geq 0$

$$\Rightarrow \left. \begin{array}{l} e_{H_d} - e_{H_P} \geq 0 \\ \text{Dar } e_{H_d} = e_{H_0} \end{array} \right\} \Rightarrow e_{H_0} - e_{H_P} \geq 0 \Rightarrow \boxed{e_{H_0} \geq e_{H_P}}$$

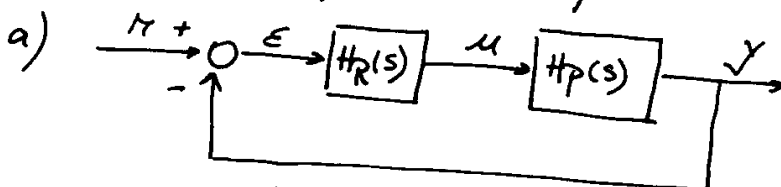
(condiție de implementabilitate a regulatorului)

Exemplu: Se dă $H_P(s) = \frac{100}{(s+2)(50s+1)}$

Se cere: a) Structura SRA

b) legea de reglare astfel încât răspunsul

sistemului să fie unu periodic cu $t_e \leq 100 \text{ ms}$, $e_{st} = 0$



$$H_P(s) = \frac{100}{(s+2)(50s+1)} = \frac{100}{2(0,5s+1)(50s+1)} = \frac{50}{(s+0,5)(50s+1)}$$

(3)

b) deoarece $\begin{cases} T_1 = 2 \leq 10 \\ T_2 = 8 \leq 10 \end{cases}$ și $T_Z = 0,1 \leq \min(T_1, T_2) = 0,2$

putem aplica criteriul modului:

$$H_R(s) = \frac{(2s+1)(8s+1)}{2 \cdot 100 \cdot 0,1s} = \frac{(2s+1)(8s+1)}{20s}$$

unde $\begin{cases} \sigma = 4,3\% < 5\% \quad \checkmark \\ \epsilon_{st} = 0 \quad \checkmark \\ t_t = 8 \cdot T_Z = 8 \cdot 0,1 = 0,8 < 1,2 \text{ sec} \quad \checkmark \\ \epsilon_n = 2 \cdot T_Z = 2 \cdot 0,1 = 0,2 < 0,3 \text{ sec} \quad \checkmark \end{cases}$

$$\begin{aligned} H_R(s) &= \frac{16s^2 + 10s + 1}{20s} = \frac{1}{20} \left(16s + 10 + \frac{1}{s} \right) = \\ &= \frac{10}{20} \left(1 + \frac{1}{10 \cdot s} + 1,6s \right) = \underbrace{\frac{1}{2}}_{k_R} \left(1 + \underbrace{\frac{1}{10s}}_{T_i} + \underbrace{1,6s}_{T_d} \right) \end{aligned}$$

$\Rightarrow H_R(s)$ este un regulator de tip PID cu:

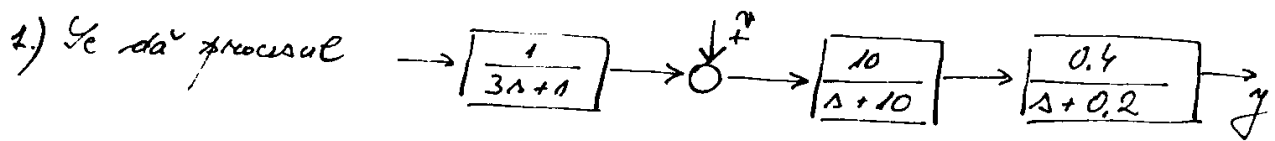
$$\begin{cases} k_R = \frac{1}{2} \\ T_i = 10 \text{ sec} \\ T_d = 1,6 \text{ sec} \end{cases}$$

$H_R(s)$ este neimplementabilă deoarece $e < 0$
($e =$ exersul de poli; $e = \text{nr. poli} - \text{nr. zerouri}$)

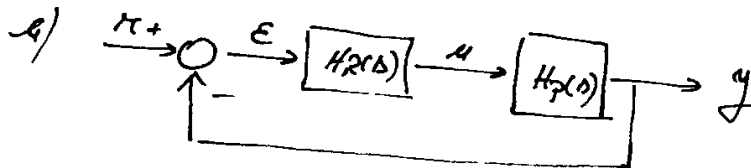
$H_R(s)$ are 2 zerouri și un pol $\Rightarrow e_{H_R} < 0$

$\Rightarrow$ pentru ca $H_R(s)$ să fie implementabilă se adaugă o filtrare pe componenta derivativă:

$$H_{RF}(s) = \frac{1}{2} \left(1 + \frac{1}{10s} + \frac{1,6s}{\alpha \cdot 1,6s + 1} \right); \text{ unde } \alpha \leq 1$$



Se cere: a) schema SRA
b) regulatorul pt care: $\left\{ \begin{array}{l} \tau \leq 5\% \\ t_e \leq 1 \text{ sec} \\ E_{st} = 0 \\ E_{ss} \leq 0.1 \end{array} \right.$



$$H_P(s) = \frac{1}{3s+1} \cdot \frac{1}{0.1s+1} \cdot \frac{2}{5s+1} = \frac{2}{(0.1s+1)(3s+1)(5s+1)}$$

b) $\left. \begin{array}{l} T_\Sigma = 0.1 \\ T_1 = 3 \\ T_2 = 5 \end{array} \right\} \text{proceso rapid cu constantă de timp parăzită}$
 $T_\Sigma \leq 0.1 \min T_k$

$\Rightarrow$ conform criteriului modului: $\left\{ \begin{array}{l} \tau \leq 4.3\% \\ t_e \leq 8 T_\Sigma = 0.8 \leq 1 \text{ sec} \\ E_{st} = 0 \\ E_{ss} = 2 T_\Sigma = 0.2 \leq 0.1 \end{array} \right.$

$\Rightarrow$ introducerea o rețea de corecție $\frac{0.1s+1}{0.05s+1} \Rightarrow$

$\Rightarrow$ noua constantă de timp parăzită este $T_\Sigma' = 0.05 \text{ sec}$

$$\Rightarrow H_R(s) \stackrel{CM}{=} \frac{(T_1 s+1)(T_2 s+1)}{2 \cdot T_\Sigma' k_R \cdot s} = \frac{(3s+1)(5s+1)}{0.2s}$$

$\Rightarrow$ regulator de tip PID ideal serie $\rightarrow T_L = 5 \text{ sec}$
 $T_d = 3 \text{ sec}$
 $k_R = 25$

$$\Rightarrow H_R(s) = 25 \cdot \frac{(3s+1)(5s+1)}{5s} \rightarrow + \text{filtrare comp. deriv}$$

$$\Rightarrow H_R(s) = 25 \cdot \frac{5s+1}{5s} \cdot \frac{3s+1}{0.01 \cdot 3s+1}, \alpha = 0.01$$

2) Se da $H_p(s) = \frac{5}{(8s+1)(s+1)(0,1s+1)}$. Se cere SRA în regulatorul pentru care $\sigma \leq 5\%$; $t_s \leq 1 \text{ sec}$; $\epsilon_M = 0$.

$$\left. \begin{array}{l} T_\Sigma = 0,1 \text{ sec} \\ T_1 = 1 \text{ sec} \\ T_2 = 8 \text{ sec} \end{array} \right\} \text{ proces rapid, } T_k \leq 10 \text{ sec, cu } T_\Sigma = 0,1$$

$$\Rightarrow \text{ref. C.M.} \left\{ \begin{array}{l} \sigma = 4,3\% \\ t_s = 8 \cdot T_\Sigma = 0,8 \leq 1 \text{ sec} \\ \epsilon_M = 0 \end{array} \right.$$

$$\Rightarrow H_R(s) = \frac{(8s+1)(s+1)}{2 \cdot 0,1 \cdot 5 \cdot s} = \frac{(8s+1)(s+1)}{s} \rightarrow \text{reg. PID serie}$$

$$\left\{ \begin{array}{l} T_i = 8 \\ T_d = 1 \\ K_R = 8 \end{array} \right. \Rightarrow H_R(s) = 8 \cdot \frac{(8s+1)(s+1)}{s}$$

$$\text{sau } H_R(s) = \frac{8s^2 + 9s + 1}{s} = 8s + 9 + \frac{1}{s} = 9 \left(1 + \frac{1}{9s} + \frac{8}{9}s \right)$$

$$\rightarrow \text{PID paralel} \left\{ \begin{array}{l} T_i = 9 \\ T_d = 8/9 \\ K_R = 9 \end{array} \right.$$

3) Se da procesul $H_p(s) = \frac{0,2}{(s+0,2)(2s+1)}$. Se cere SRA în regulatorul pt care $\sigma \leq 5\%$; $t_s \leq 1 \text{ sec}$; $\epsilon_M = 0$.

$$H_p(s) = \frac{1}{(5s+1)(2s+1)} \quad e_p = 2 \Rightarrow \text{aligem } e_{H_0} = 2$$

$$\Rightarrow H_0(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{aligem } \zeta = 0,7 \Rightarrow \sigma = 4,3\%$$

$$\zeta \in (0,6; 0,8) \Rightarrow t_s \cong \frac{4}{\zeta\omega_n} = \frac{4}{0,7\omega_n} \leq 1 \text{ sec} \Rightarrow \omega_n \geq \frac{4}{\zeta}$$

$$\Rightarrow \omega_n \geq \frac{4}{0,7} \Rightarrow \omega_n \geq 5,7 \Rightarrow$$

$$H_d(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}; \quad H_R(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \cdot (5s+1)(2s+1)$$

$$\Rightarrow H_R(s) = \frac{\omega_n^2}{2\zeta s \left(\frac{1}{2\zeta\omega_n} s + 1 \right)} (5s+1)(2s+1)$$

$$\begin{aligned} \bar{I} \cdot \frac{1}{2\zeta\omega_n} &= 5 \Rightarrow \zeta\omega_n = 0,1 \Rightarrow \omega_n = \frac{0,1}{0,7} \text{ NU}; \omega_n = \frac{1}{10\zeta} < \frac{4}{\zeta} \\ \bar{I} \cdot \frac{1}{2\zeta\omega_n} &= 2 \Rightarrow \zeta\omega_n = 0,25 \Rightarrow \omega_n = \frac{0,25}{0,7} \text{ NU}; \omega_n = \frac{1}{4\zeta} < \frac{4}{\zeta} \end{aligned}$$

$$\Rightarrow \text{valigem } \omega_n = 6$$

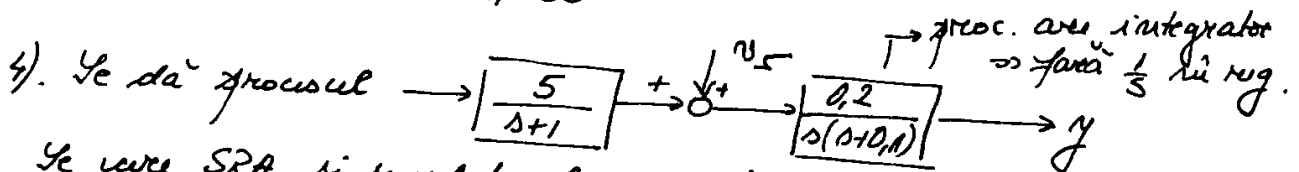
$$\Rightarrow H_R(s) = \frac{6}{1,4\Delta(0,11s)} \cdot (50M)(20M) = \frac{6}{1,4} \cdot \frac{100^2 + 70M}{\Delta} \cdot \frac{1}{0,11s}$$

$$= \frac{6}{1,4} \cdot \left(\frac{100 + 7 + 1}{\Delta} \right) \cdot \frac{1}{0,11s} = \frac{6}{1,4} \cdot 7 \left(1 + \frac{1}{70} + \frac{100}{7} \right) \cdot \frac{1}{0,11s}$$

$$\left. \begin{array}{l} T_L = 5 \\ T_d = 2 \end{array} \right\} \Rightarrow H_R(s) = \frac{6}{1,4} \cdot \frac{(50M)(20M)}{5 \cdot \Delta} \cdot \frac{5}{0,11s} \Rightarrow k_R = \frac{30}{1,4}$$

$$0,11 = \alpha \cdot T_d = \alpha \cdot 2 \Rightarrow \alpha = \frac{0,11}{2} = 0,055$$

$$\Rightarrow \text{reg PIDT}, \left. \begin{array}{l} T_L = 5 \\ T_d = 2 \\ k_R = 30/1,4 \\ \alpha = 0,055 \end{array} \right\}$$



Se cere SRA și regulatorul care îndeplinește condițiile:
 $\sigma \leq 5\%$; $t_r \leq 2\text{ms}$; $\epsilon_{st} = 0$; $\epsilon_{ss} \leq 0,2\text{ms}$

$$H_P(s) = \frac{5}{s+1} \cdot \frac{2}{s(100s)} = \frac{10}{s(s+1)(100s)} \rightarrow \text{proces rapid}$$

$$\left. \begin{array}{l} T_L = 1\text{ms} \\ T_1 = 10\text{ms} \end{array} \right\} \Rightarrow \text{ref. CM.} \quad \left. \begin{array}{l} \sigma = 4,3\% \\ t_r = 8T_L = 8\text{ms} \neq 2\text{ms} \\ \epsilon_{st} = 0 \\ \epsilon_{ss} = 2T_L = 2\text{ms} \neq 0,2\text{ms} \end{array} \right\}$$

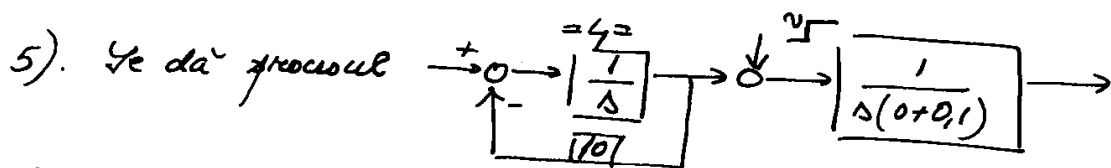
$\Rightarrow$ alegem $T_L' = 0,1\text{ms} \Rightarrow$ introducem o rețea de derivate

$$\text{Alina } H_{\text{Alina}}(s) = \frac{s+1}{0,1s} \Rightarrow t_r = 8 \cdot T_L' = 0,8 \leq 2\text{ms}$$

$$\epsilon_{ss} = 2 \cdot T_L' = 0,2 \leq 0,2\text{ms}$$

$$\Rightarrow H_R(s) = \frac{100s}{2 \cdot T_L' \cdot k_P} = \frac{100s+1}{2} = \frac{1}{2} (1 + 100s) \rightarrow \text{reg PD}$$

$$\neq \text{impuls} \Rightarrow \text{fără derivă} \Rightarrow H_R(s) = \frac{1}{2} \left(1 + \frac{100}{\alpha \cdot 100s} \right) \mid \alpha = 0,01$$



Se cere SRA și regulatorul cu $\sigma \leq 5\%$; $t_E \leq 1 \text{ ms}$; $E_{st} = 0$; $E_N \leq 0,1 \text{ ms}$.

$$H_p(s) = \frac{1}{1 + \frac{10}{s}} \cdot \frac{1}{s \cdot 0,1(10s+1)} = \frac{1}{s+10} \cdot \frac{10}{s(10s+1)}$$

$$= \frac{0,1}{(0,1s+1) \cdot s \cdot (10s+1)} = \frac{1}{s(0,1s+1)(10s+1)} \rightarrow \text{proces rapid} \left\{ \begin{array}{l} T_Z = 0,1 \\ T_1 = 10 \end{array} \right.$$

$\Rightarrow$ cf. CM.

$$\left\{ \begin{array}{l} \sigma = 4,3\% \\ t_E = 8 T_Z = 0,8 \leq 1 \text{ ms} \checkmark \\ E_{st} = 0 \\ E_N = 2 \cdot T_Z = 0,2 \neq 0,1 \text{ ms} \end{array} \right.$$

$\Rightarrow T_Z' = 0,05 \Rightarrow$ introducerea o rețea de corecție $H_{rețea}(s) = \frac{0,1s}{0,05}$

$\Rightarrow E_N = 2 \cdot T_Z' = 0,1 \leq 0,1 \text{ ms} \checkmark$

$\Rightarrow H_R(s) = \frac{10s+1}{0,1} = 10(1+10s) \rightarrow \text{reg. PD}$

$\Rightarrow \text{PD} \bar{T} : H_R(s) = 10 \left(1 + \frac{100}{\alpha \cdot 10s} \right), \alpha = 0,01$

6) Se dă procesul $H_p(s) = \frac{2}{8s(0,1s+1)}$. Se cere alg. de comandă cu $E_N = 0$.

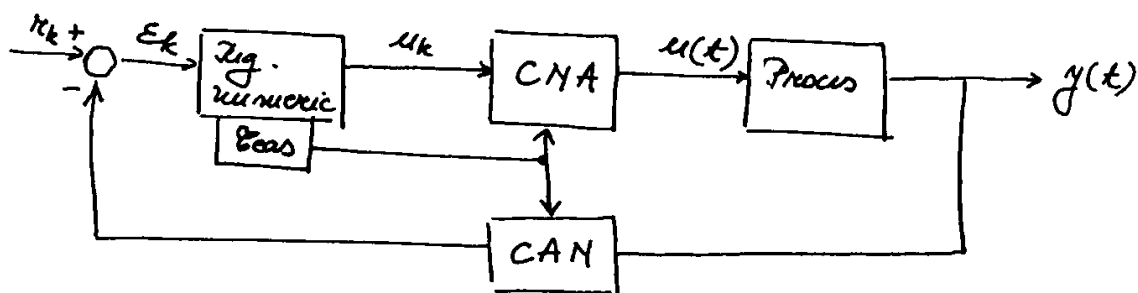
Sunt impuse condițiile de aplicare a criteriului similitudinii: $T_K = 8 \text{ ms}$; $T_Z = 0,1 \text{ ms}$; $K_P = 2$

Atunci: $H_R(s) = \frac{4 T_Z s + 1}{8 T_Z^2 K_P s} = \frac{4 \cdot 0,1 s + 1}{8 \cdot 0,01 \cdot 2 s} = \frac{0,4s + 1}{0,16s} \rightarrow \text{reg. PI}$

$T_L = 0,4$; $K_T = 10/4$: $H_R(s) = 10/4 \cdot \frac{0,4s + 1}{0,4s}$

$E_N = 0$ deoarece sistemul continuu $\frac{1}{s^2}$. $E_N = \lim_{s \rightarrow 0} s \cdot \frac{1}{1+H_R(s)} \cdot \frac{1}{s} = 0$.

SISTEME DISCRETE



$$u(t) = u_k \text{ for } t \in [kT, (k+1)T]$$

$$H_{eoz} = \frac{1 - e^{-sT}}{s} \quad (\text{extrapolator de ordinu zero})$$

Alegerea perioadei de discretizare (T) se face în funcție de:

- particularitățile procesului
- gama perturbărilor
- performanțele cerute

- $T \in (0,1 \div 0,3) T_i$ și $\arg P_i$
- $T \approx (0,3 \div 1,1) T_0$
- $T \in (0,1 \div 0,3) T_0$ din metoda Ziegler-Nichols
- $T \in \left(\frac{1}{6} \div \frac{1}{15}\right) t_L$

Discretizarea procesului: $H_C(z^{-1}) = \mathcal{Z} \left\{ H_{eoz}(s) \cdot H_p(s) \right\}$

$$\Rightarrow \left[H_C(z^{-1}) = (1 - z^{-1}) \mathcal{Z} \left\{ \frac{H_p(s)}{s} \right\} \right]$$

$$\mathcal{Z} \left\{ e^{-s\tau} \right\} = z^{-\frac{\tau}{T}}$$

$$\tau = d \cdot T; \quad d \in \mathbb{N}$$

$$\mathcal{Z} \left\{ \frac{1}{s} \right\} = \frac{1}{1 - z^{-1}}$$

$$\mathcal{Z} \left\{ \frac{1}{s+a} \right\} = \frac{1}{1 - z^{-1} e^{-\frac{T}{a}}}$$

=2=

Exemplu: Date: - $H_p(s)$

- răsp. impuls $y = \sum_{l=0}^k y(lT) z^{-l}$ cu $\Delta T = 1$

$$\Rightarrow H_0 = \frac{y}{x}$$

$$H_c(z^{-1}) = (1 - z^{-1}) z \left\{ \frac{H_p(s)}{\Delta} \right\}$$

$$H_R(z^{-1}) = \frac{H_0}{1 - H_0} \cdot \frac{1}{H_c}$$

1). $H_p(s) = \frac{0,5 e^{-2s}}{10s + 1}$

a) SRA cu RN

b) echivalentul discret valgaud T

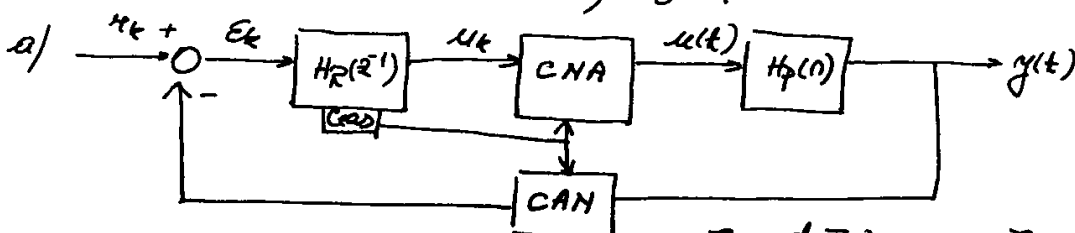
c) AN ai $y(0) = y(T) = y(2T) = 0$

$$y(3T) = 0,4; y(4T) = 0,9$$

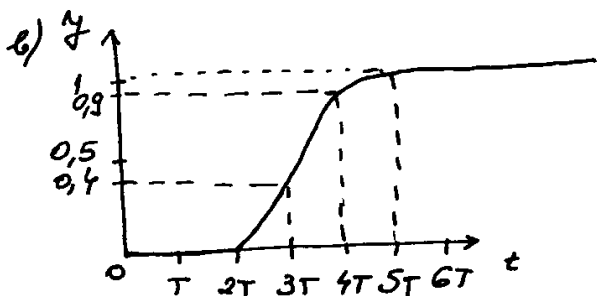
$$y(5T) = y(6T) = \dots = 1$$

d) structura calc. în vederea implementării

e) $u_0 = ?$



$$\left. \begin{array}{l} b = d \cdot T \\ d = 2 \end{array} \right\} \Rightarrow T = \frac{b}{d} = 1$$



$$H_c(z^{-1}) = (1 - z^{-1}) \cdot z \left\{ \frac{H_p(s)}{\Delta} \right\} = (1 - z^{-1}) \cdot z \left\{ \frac{0,5 e^{-2s}}{\Delta(10s + 1)} \right\}$$

$$z \left\{ \frac{0,5 e^{-2s}}{\Delta(10s + 1)} \right\} = 0,5 z^{-\frac{2}{T}} z \left\{ \frac{1}{\Delta(10s + 1)} \right\}$$

$$\frac{1}{\Delta(10s + 1)} = \frac{A}{\Delta} + \frac{B}{10s + 1} = \frac{10As + A + B\Delta}{\Delta(10s + 1)} \Rightarrow \begin{cases} 10A + B = 0 \\ A = 1 \Rightarrow B = -10 \end{cases}$$

$$\begin{aligned} & \Rightarrow \mathcal{Z} \left\{ \frac{1}{\Delta(10\Delta+1)} \right\} = \mathcal{Z} \left\{ \frac{1}{\Delta} \right\} + \mathcal{Z} \left\{ \frac{-10}{10\Delta+1} \right\} = \mathcal{Z} \left\{ \frac{1}{\Delta} \right\} - \mathcal{Z} \left\{ \frac{1}{\Delta + \frac{1}{10}} \right\} \\ & = \frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1}e^{-j/10}} \end{aligned}$$

$$\begin{aligned} \Rightarrow H_C(z^{-1}) &= (1-z^{-1}) \cdot 0.5 \cdot z^{-2} \left(\frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1}e^{-j/10}} \right) = \\ &= \frac{(1-z^{-1}) \cdot 0.5 \cdot z^{-2} \cdot (1-z^{-1}e^{-j/10})}{(1-z^{-1})(1-z^{-1}e^{-j/10})} = \frac{0.5 z^{-2} \cdot z^{-1} (1-e^{-j/10})}{1-z^{-1}e^{-j/10}} = \frac{0.5 z^{-3} \cdot 1}{1-z^{-1} \cdot 0.9} \end{aligned}$$

$$H_C(z^{-1}) = \frac{0.5 z^{-3}}{1-0.9 z^{-1}}$$

$$\begin{aligned} c) \gamma(\vec{z}) &= \gamma(0) \cdot z^0 + \gamma(1) \cdot z^{-1} + \gamma(2) \cdot z^{-2} + \gamma(3) \cdot z^{-3} + \gamma(4) \cdot z^{-4} + \dots = \\ &= 0.4 z^{-3} + 0.9 z^{-4} + z^{-5} + z^{-6} + \dots = \\ &= 0.4 z^{-3} + 0.9 z^{-4} + z^{-5} (1 + z^{-1} + z^{-2} + \dots) = \\ &= 0.4 z^{-3} + 0.9 z^{-4} + z^{-5} \cdot \frac{1}{1-z^{-1}} = \frac{0.4 z^{-3} - 0.4 z^{-4} + 0.9 z^{-4} - 0.9 z^{-5} + z^{-5}}{1-z^{-1}} = \\ &= \frac{0.4 z^{-3} + 0.5 z^{-4} + 0.1 z^{-5}}{1-z^{-1}} \end{aligned}$$

$$H_0(z^{-1}) = \frac{\gamma(\vec{z})}{\kappa(\vec{z})} \Rightarrow H_0(z^{-1}) = 0.4 z^{-3} + 0.5 z^{-4} + 0.1 z^{-5}$$

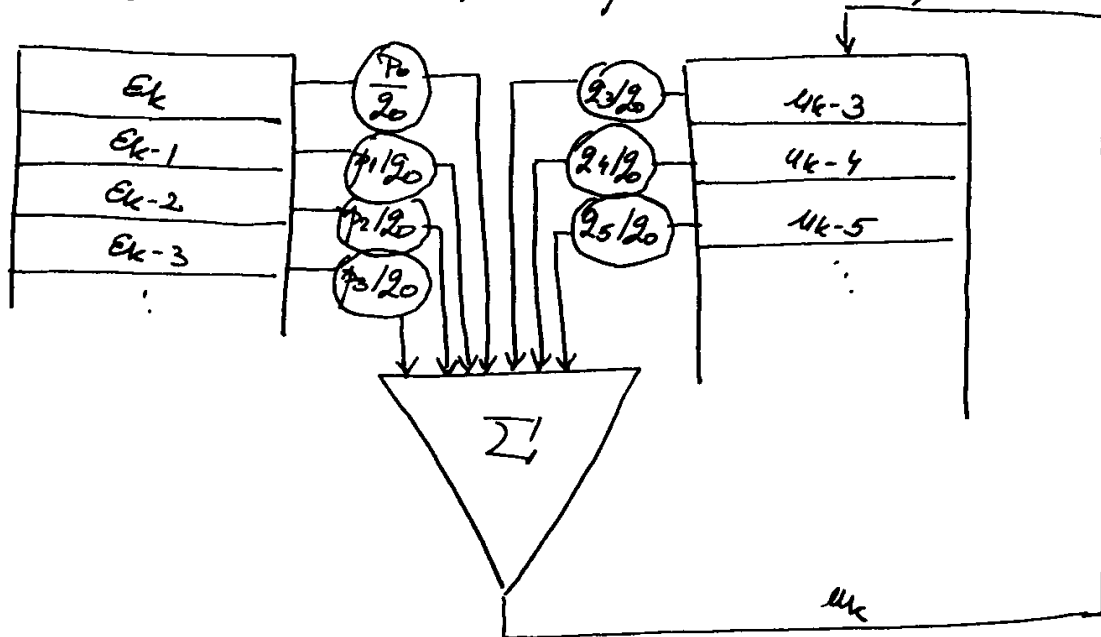
$$\kappa(\vec{z}) = \mathcal{Z} \left\{ \frac{1}{\Delta} \right\} = \frac{1}{1-z^{-1}}$$

$$\Rightarrow H_P(z^{-1}) = \frac{H_0}{1-H_0} \cdot \frac{1}{H_C} = \frac{0.4 z^{-3} + 0.5 z^{-4} + 0.1 z^{-5}}{1 - 0.4 z^{-3} - 0.5 z^{-4} - 0.1 z^{-5}} \cdot \frac{1 - 0.9 z^{-1}}{0.5 z^{-3}} =$$

$$= \frac{z^{-3} (0.4 + 0.5 z^{-1} + 0.1 z^{-2}) (1 - 0.9 z^{-1})}{0.5 z^{-3} (1 - 0.4 z^{-3} - 0.5 z^{-4} - 0.1 z^{-5})} = \frac{z_0 + z_1 z^{-1} + z_2 z^{-2} + z_3 z^{-3}}{z_0 + z_3 z^{-3} + z_4 z^{-4} + z_5 z^{-5}}$$

$$H_P(z^{-1}) = \frac{U(z^{-1})}{E(z^{-1})} \Leftrightarrow z_0 \cdot u_k + z_3 u_{k-3} + z_4 u_{k-4} + z_5 u_{k-5} = z_0 e_k + z_1 e_{k-1} + z_2 e_{k-2} + z_3 e_{k-3}$$

$$\Rightarrow u_k = \frac{1}{20} (70 E_k + 71 E_{k-1} + 72 E_{k-2} + 73 E_{k-3} - 23 u_{k-3} - 24 u_{k-4} - 25 u_{k-5})$$



e) $u_0 = \frac{70}{20} E_0 = \frac{70}{20} (x_0 - y_0) = \frac{70}{20} (1 - 0) = \frac{70}{20} = \frac{0,7}{0,5} = 0,8$

2) $H_p(s) = \frac{10}{(s+10)(100\pi)}$

a) SRA cu RN

b) echiv. direct $T=1$ si $T=2000$

c) alg. numeric ai' $y(0)=0$; $y(T)=0,5$;
 $y(2T)=0,9$; $y(3)=y(4)=$

d) $u_0 = ?$

e) implementare

6) $H_C(z^{-1}) = (1-z^{-1}) z \left\{ \frac{H_p(s)}{s} \right\} = (1-z^{-1}) \cdot z \left\{ \frac{10}{(s+10)(100\pi)s} \right\}$

$$\frac{10}{s(s+10)(100\pi)} = \frac{1}{(10,10)(100\pi) \cdot s}$$

neglijăm

$$z \left\{ \frac{1}{s(100\pi)} \right\} = z \left\{ \frac{1}{s} \right\} - z \left\{ \frac{10}{1+100} \right\} = z \left\{ \frac{1}{s} \right\} - z \left\{ \frac{1}{s+\frac{1}{10}} \right\}$$

$$= \frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1}e^{-T/10}} \Rightarrow H_C(z^{-1}) = \frac{1-z^{-1}e^{-T/10}}{(1-z^{-1})(1-z^{-1}e^{-T/10})}$$

$$\Rightarrow H_C(z^{-1}) = \frac{z^{-1}(1 - e^{-T/10})}{1 - z^{-1}e^{-T/10}} \stackrel{T=5}{=}$$

$$\boxed{T=1} \Rightarrow H_C(z^{-1}) = \frac{z^{-1}(1 - e^{-1/10})}{1 - z^{-1}e^{-1/10}} = \frac{0,1z^{-1}}{1 - 0,9z^{-1}}$$

$$\boxed{T=2} \Rightarrow H_C(z^{-1}) = \frac{z^{-1}(1 - e^{-2/10})}{1 - z^{-1}e^{-2/10}} = \frac{0,19z^{-1}}{1 - 0,819z^{-1}}$$

$$\begin{aligned} \text{b) } y(z^{-1}) &= 0,5z^{-1} + 0,9z^{-2} + z^{-3} + z^{-4} + \dots = \\ &= 0,5z^{-1} + 0,9z^{-2} + z^{-3}(1 + z^{-1} + z^{-2} + \dots) = \\ &= 0,5z^{-1} + 0,9z^{-2} + \frac{z^{-3}}{1 - z^{-1}} = \frac{0,5z^{-1} - 0,5z^{-2} + 0,9z^{-2} - 0,9z^{-3} + z^{-3}}{1 - z^{-1}} \\ &= \frac{0,5z^{-1} + 0,4z^{-2} + 0,1z^{-3}}{1 - z^{-1}} \end{aligned}$$

$$\Rightarrow H_0(z^{-1}) = \frac{y(z^{-1})}{x(z^{-1})} = 0,5z^{-1} + 0,4z^{-2} + 0,1z^{-3}$$

$$\Rightarrow H_R(z^{-1}) = \frac{H_0}{1 - H_0} \cdot \frac{1}{H_C} = \frac{0,5z^{-1} + 0,4z^{-2} + 0,1z^{-3}}{1 - 0,5z^{-1} - 0,4z^{-2} - 0,1z^{-3}} \cdot \frac{1 - 0,9z^{-1}}{0,1z^{-1}} \quad \uparrow \quad \overline{T=1}$$

$$\Rightarrow H_R(z^{-1}) = \frac{(0,5z^{-1} + 0,4z^{-2} + 0,1z^{-3})(1 - 0,9z^{-1})}{0,1z^{-1}(1 - 0,5z^{-1} - 0,4z^{-2} - 0,1z^{-3})} = \frac{z_0 + z_1z^{-1} + z_2z^{-2} + z_3z^{-3}}{z_0 + z_1z^{-1} + z_2z^{-2} + z_3z^{-3}}$$

$$H_R(z^{-1}) = \frac{U(z^{-1})}{C(z^{-1})} \Rightarrow u_k = \frac{1}{z_0} (z_0 + z_1z^{-1} + z_2z^{-2} + z_3z^{-3} - z_0z^{-1} - z_1z^{-2} - z_2z^{-3} - z_3z^{-4})$$

$$\text{d) } u_0 = \frac{z_0}{z_0} = \frac{0,5}{0,1} = 5$$

$$H_R(z^{-1}) = \frac{5 - 0,5z^{-1} - 2,6z^{-2} - 0,9z^{-3}}{1 - 0,5z^{-1} - 0,4z^{-2} - 0,1z^{-3}}$$

$$3) H_p(z) = \frac{20 e^{-8\Delta}}{(\Delta+20)(20\Delta+1)}$$

= 6 =
a) SRA cu RN

b) dezvolt. în părți continue alegând corect
c) alg. de reg. numerică ai: $y(0) = y(1) = \dots = y$

$$\begin{cases} y(5T) = 0,2 \\ y(6T) = 1 \end{cases}$$

d) val. convergenței inițiale în funcție de T

e) implementare

$$b) H_c(z^{-1}) = (1-z^{-1}) \cdot \left\{ \frac{H_p(z)}{\Delta} \right\}, \quad \boxed{T = 4T} \Rightarrow \frac{T}{T} = 4 \Rightarrow \boxed{T = 2}$$

$$\frac{20 e^{-8\Delta}}{(\Delta+20)(20\Delta+1)} = \frac{20 e^{-8\Delta}}{20(0,05\Delta+1)(20\Delta+1)}$$

↳ se neglijează $0,05 \Delta < 20$

$$H_c(z^{-1}) = (1-z^{-1}) \cdot \left\{ \frac{20 e^{-8\Delta}}{20(20\Delta+1) \cdot \Delta} \right\} = (1-z^{-1}) \cdot \left\{ \frac{e^{-8\Delta}}{(20\Delta+1)\Delta} \right\}$$

$$\begin{aligned} H_c(z^{-1}) &= (1-z^{-1}) \cdot \frac{1}{20} \cdot z^{-4} \cdot \left\{ \frac{1}{20(\Delta + \frac{1}{20})\Delta} \right\} = (1-z^{-1}) \cdot z^{-4} \cdot \left(\frac{1}{1-z^{-1}e^{-T/20}} - \frac{1}{1-z^{-1}e^{-T}} \right) \\ &= (1-z^{-1}) \cdot \frac{1}{200} \cdot z^{-4} \cdot \frac{1}{1-z^{-1}e^{-T/20}} = \frac{1-z^{-1}}{200(1-z^{-1}e^{-T/20})} \cdot z^{-4} \end{aligned}$$

$$\begin{aligned} c) y(z^{-1}) &= 0,8z^{-5} + z^{-6} + z^{-7} + \dots = 0,8z^{-5} + z^{-6}(1 + z^{-1} + z^{-2} + \dots) \\ &= 0,8z^{-5} + \frac{z^{-6}}{1-z^{-1}} = \frac{0,8z^{-5} - 0,8z^{-6} + z^{-6}}{1-z^{-1}} = \frac{0,8z^{-5} + 0,2z^{-6}}{1-z^{-1}} \end{aligned}$$

$$H_0(z^{-1}) = \frac{y(z^{-1})}{x(z^{-1})} = 0,8z^{-5} + 0,2z^{-6}$$

$$H_R(z^{-1}) = \frac{H_0}{1-H_0} \cdot \frac{1}{H_c} = \frac{0,8z^{-5} + 0,2z^{-6}}{1 - 0,8z^{-5} - 0,2z^{-6}} \cdot \frac{200(1-z^{-1}e^{-T/20})}{(1-z^{-1}) \cdot z^{-4}}$$

$$= \frac{200(0,8z^{-1} + 0,2z^{-2})}{1 - 0,8z^{-5} - 0,2z^{-6}} \cdot \frac{1-z^{-1}e^{-T/20}}{z^5(1-e^{-T/20})} =$$

$$= \frac{(0,8 + 0,2z^{-1})(1-z^{-1}e^{-T/20})}{z^5(1 - 0,8z^{-5} - 0,2z^{-6})(1-e^{-T/20})}$$